Optimization of Volume-to-Point Heat Conduction Problem

Application of entransy theory

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Volume-to-Point Heat Conduction Problem

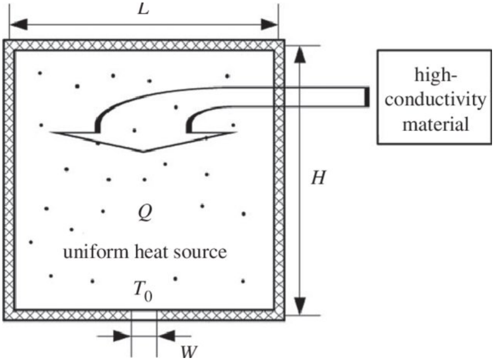


Fig. 1: 2D heat conduction with a uniformly distributed internel heat source.

- Uniformly heat source $\dot{q}_s$ in the domain.
- Only one low temperature point at boundary T_0.
- $\int\int\int k \mathrm{d}V = \text{const.}$

How to distribute high conductivity material to make the average temperature of the domain lowest?

Principle of Minimum Entransy Dissipation

Minimum

$$\int k(\nabla T)^2 \mathrm{d}V$$

under the constraint

$$\int k \mathrm{d}V = \text{const.}$$

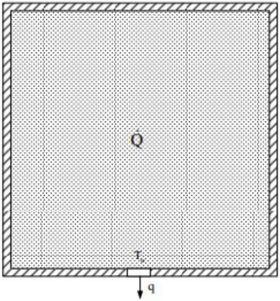
By Lagrange multiplier, it can be derived that the optimum condition should satisfy

$$(\nabla T)^2 = \text{const}$$

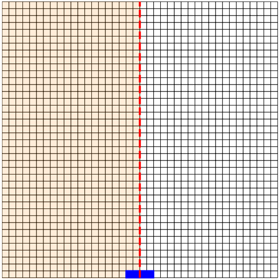
The temperature gradient of the domain should be as uniform as possible!

1. Introduction
2. Simulation Details
    - Problem Statement
    - Numerical Scheme
    - Relaxation factor and Grid Independence Study
    - Optimization Algorithm
3. Results and Discussion

Problem Statement



(a) The schematic diagram.



(b) Solving domain and mesh discretization.

- 10% heat flux boundary.
- Only the left side is solved.

Finite Volume Method

- Interior node method.
- Interfacial thermal conductivity - harmonic average:

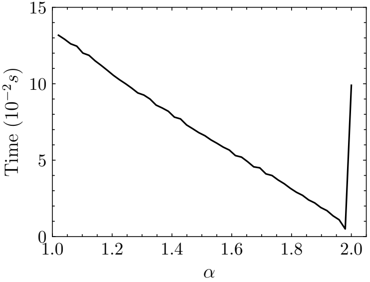
$$\lambda_{\text{ave}} = \frac{2\lambda_{\text{high}}\lambda_{\text{low}}}{\lambda_{\text{high}} + \lambda_{\text{low}}}$$

- SOR iteration scheme:

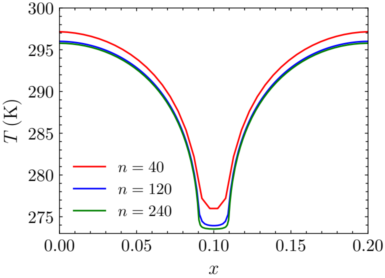
$$\begin{aligned} T_P^{(n)} = T_P^{(n-1)} \\ + \alpha \left(\frac{a_W T_W^{(n)} + a_S T_S^{(n)} + a_E T_E^{(n-1)} + a_N T_N^{(n-1)} + b}{a_P} \right) \\ - \alpha \left(T_P^{(n-1)} \right) \end{aligned}$$

α is the relaxation factor.

Relaxation factor and Grid Independence Study



(a) Computation time with different relaxation factor.



(b) Temperature distributions along the heat outlet plane.

Optimization Algorithm

Fill point by point

1. fill in the domain with substrate.
2. determine T distribution by solving energy equation.
3. put high k where ∇T is larget.
4. return (2) until high k meterial is used up.

Integrative optimization

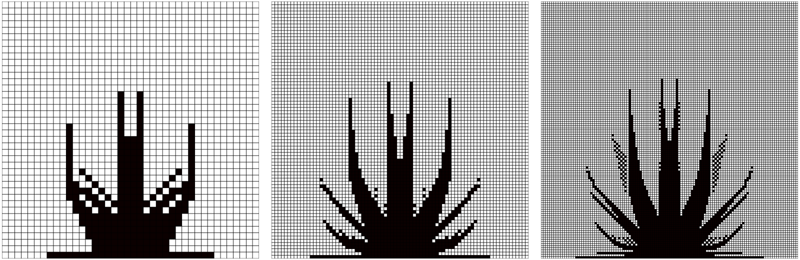
5. take away high k material iteratively, resolve T distribution, find the least effective high k position, refill the high k material to other places.

1 Introduction

2 Simulation Details

3 Results and Discussion
   Results Under Low k_h/k_l
   Results Under High k_h/k_l

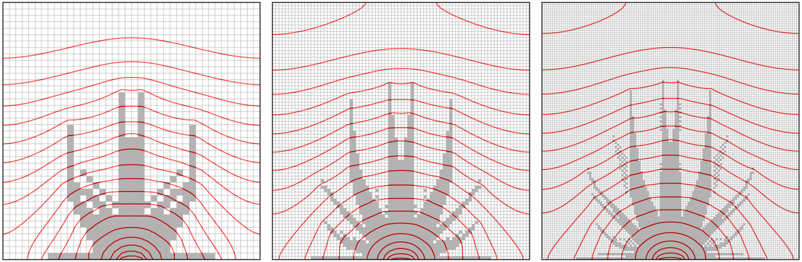
Grid Size Dependence Analysis



(c) $n = 40$ (d) $n = 80$ (e) $n = 120$

Fig. 2: High k material distributions with $k_h/k_l = 5, \phi_r = 0.15$.

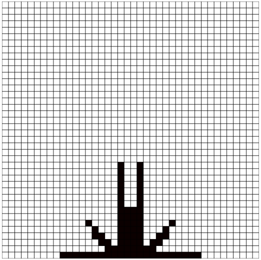
Grid Size Dependence Analysis



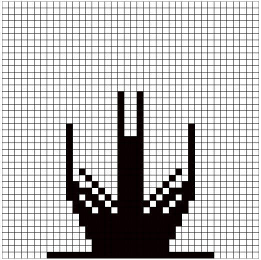
(a) $n = 40$ (b) $n = 80$ (c) $n = 120$

Fig. 3: Temperature distribution with $k_h/k_l = 5, \phi_r = 0.15$.

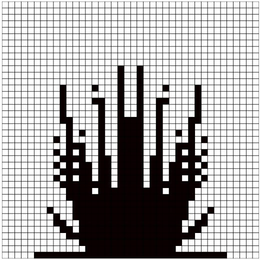
Results with Different Filling Rates



(a) $\phi_r = 0.05$



(b) $\phi_r = 0.15$



(c) $\phi_r = 0.25$

Fig. 4: High k material distribution with $n = 40, k_h/k_l = 5$.

Filling Process

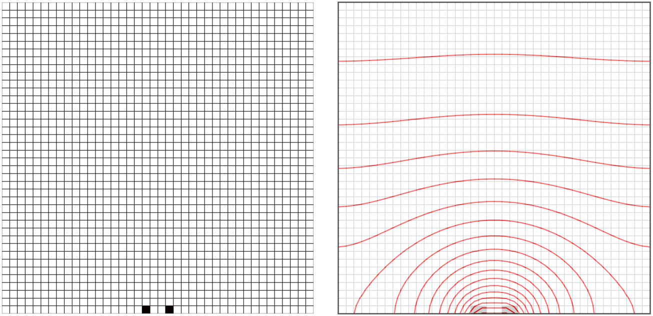
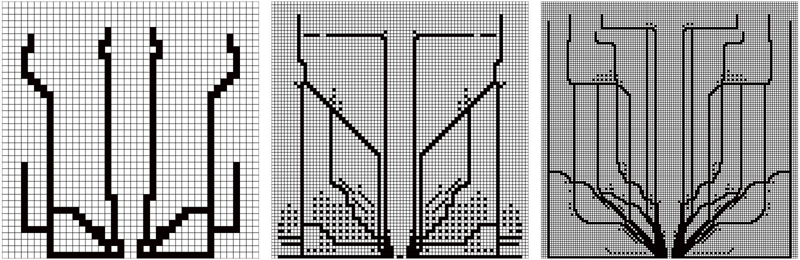


Fig. 5: High k material filling process with $n = 40$, $k_h/k_l = 5, \phi_r = 0.15$.

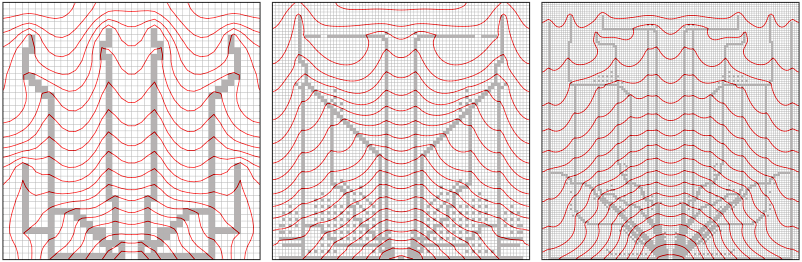
Grid Size Dependence Analysis



(a) $n = 40$ (b) $n = 80$ (c) $n = 120$

Fig. 6: High k material distributions with $k_h/k_l = 100, \phi_r = 0.15$.

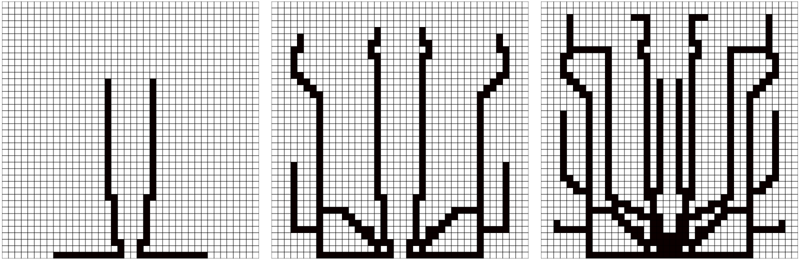
Grid Size Dependence Analysis



(a) $n = 40$ (b) $n = 80$ (c) $n = 120$

Fig. 7: Temperature distribution with $k_h/k_l = 100, \phi_r = 0.15$.

Results with Different Filling Rates



(a) $\phi_r = 0.05$ (b) $\phi_r = 0.15$ (c) $\phi_r = 0.25$

Fig. 8: High k material distribuions with $n = 40, k_h/k_l = 100$.

Filling Process

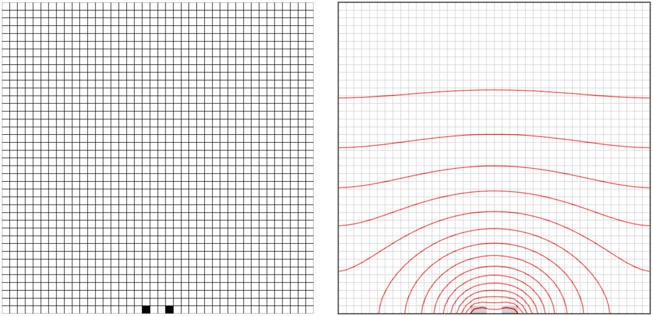


Fig. 9: High k material filling process with $n = 40, k_h/k_l = 100, \phi_r = 0.15$.

Thank You!